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A 3-D coupled surface-subsurface model to investigate the runoff dynamics on hillslopes

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Abstract: The numerous field and laboratory studies that have been conducted on hillslopes have permitted the identification of various processes that explain hillslope runoff and the contribution of rapid subsurface flow during rainfall events. But, if some theories are now well accepted (i.e. the saturated contributing area theory in the temperate climate), no general and comprehensive view of hillslope response during rainfall events is available. We argue that detailed numerical models will be very helpful in building this comprehensive view. The possible coexistence, on hillslopes, of interacting surface and subsurface flows induces specific numerical constraints. We present a 3-D coupled numerical model where the Richards equations in the subsurface domain and the diffusive wave equations on the surface are solved simultaneously. Using this framework, numerical investigations are conducted on simplified hillslope transects to analyse the processes activation, and to identify the major control factors. The first results show that various processes described in the literature (water table ridging, saturated contributing areas, pressure wave translatory flow) are simultaneously active even for a simplified and homogeneous hillslope. Moreover, exfiltration is often a major source of runoff in the saturated contributing areas. The results are compared to the ones obtained with the simplified hillslope storage Boussinesq model (hsB) proposed by Troch et al. [4]. This comparison reveals the important role played by the unsaturated zones in the dynamics of hillslope response during floods.

I. Description of the numerical model

Subsurface flow : Richards equation

$$\frac{\partial}{\partial t}\left(\rho\phi \mathrm{S_l}(p_c)\right) + \nabla\cdot\left[-\frac{\rho k_{\mathrm{int}}}{\mu}\bar{\bar{\mathbf{t}}}_{\mathbf{k}}\, \mathrm{k_{rel}}(p_c)\nabla H\right] = S \qquad (1)$$

with p_c, the capillary pressure; $H = z - \frac{p_c}{\rho g}$, the total pressure head ; $\mathrm{S_l}(p_c)$, the saturation function and $\mathrm{k_{rel}}(p_c)$, the relative conductivity function. S is a sink/source term taking in account rainfall and water exchange with the soil surface.

Overland flow : diffusive wave equation

$$\frac{\partial}{\partial t}\left(\rho \mathrm{S_c}(y)\right) + \nabla\cdot\left[-\frac{\rho}{\mu}\bar{\bar{\mathbf{t}}}_{\mathbf{n}}\Phi^{-\frac{1}{2}}\mathrm{A}(y)\mathrm{R_H}(y)^{\frac{2}{3}}\nabla H\right] = S \qquad (2)$$

with y, the surface water depth; $H = y + z$, the total pressure head; $\mathrm{S_c}(y)$, the depression storage function; $\mathrm{A}(y)$ the flow obstruction function; $\mathrm{R_H}$, the hydraulic radius and $\Phi = \sqrt{\sum_i (\partial H/\partial x_i)^2}$.

Discretization and coupling method

Since equation (1) and (2) are written using the form $\frac{\partial}{\partial t}M(u) + \nabla\cdot[K_o(u)\nabla H] = S$, the control volume finite element method [2, 5] is applied in both domains. Assuming the continuity of the pressure field in the whole domain, equations are coupled by substitution of the exchange terms S.

Old-event water model

Hydrograph composition is computed using a simple mixing-advection model:

$$\frac{\partial}{\partial t}(M(u)\cdot\chi) + \nabla\cdot[K_o(u)\cdot\nabla H\cdot\chi] = S_\chi \qquad (3)$$

II. The simplified hillslope

The considered simplified hillslope is non-convergent, homogeneous, 50 meters long, with a soil depth of 2 meters and a constant slope of 10%. The soil is sandy ($K_S = 1$ m/h) [3].

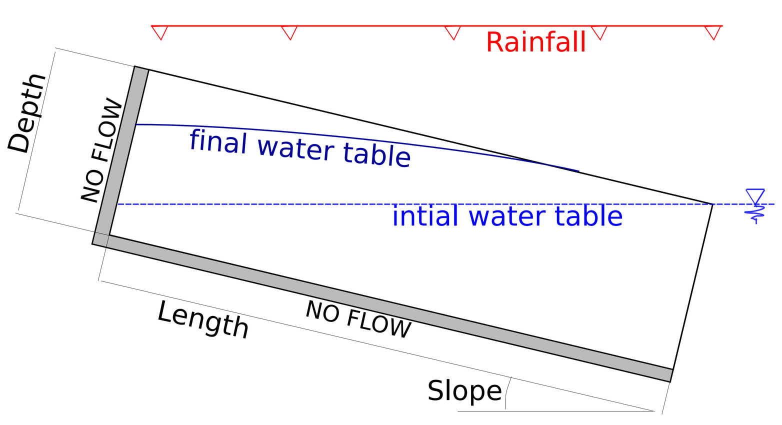

Fig. 1. Geometry of the hillslope

III. Behaviour of the simplified hillslope

The following results illustrate the evolution in time of the water table and the downstream outflow discharges when the hillslope receives a constant rainfall of 30 mm/h. A zero-flow condition is applied here at the downstream boundary: symmetry condition [1].
The equilibrium is reached after $Te \approx 70\,h$.

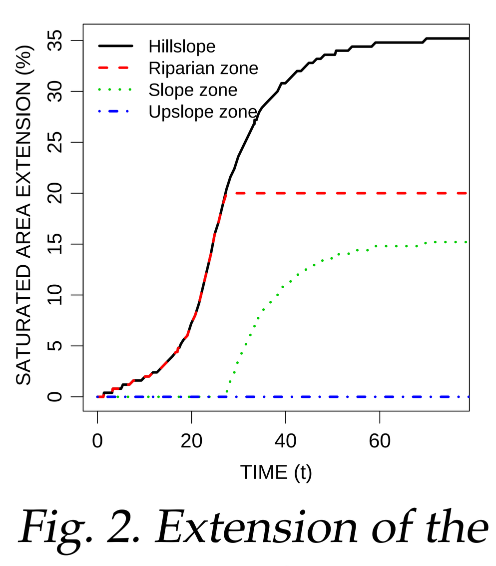

Fig. 2. Extension of the saturated areas

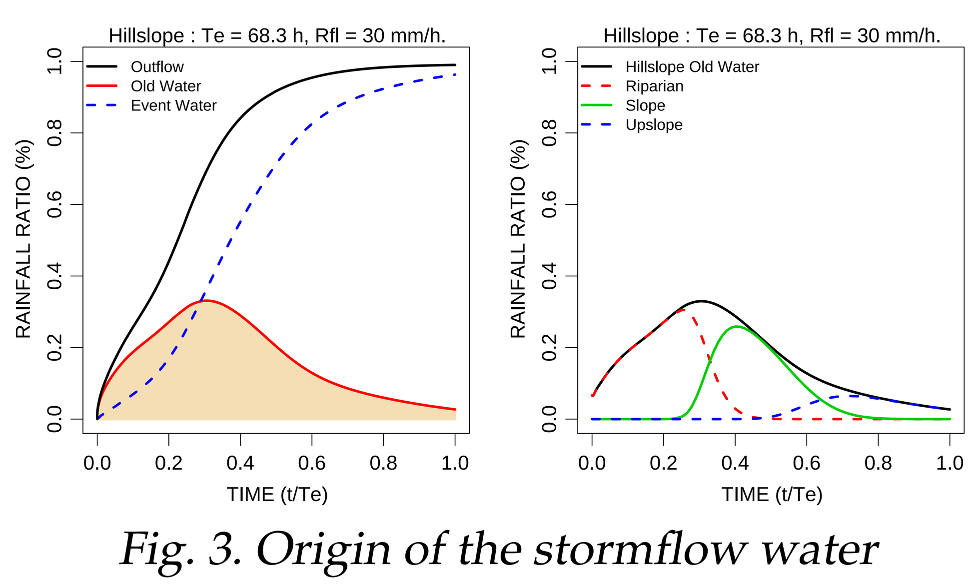

Fig. 3. Origin of the stormflow water

The model simulates a progressive extension of the saturated areas along the hillslope. After 10 hours ($t/Te = 0.14$, 300 mm rainfall) the total runoff coefficient is close to 30% (see Fig. 3-a) but only 2-3% of the hillslope is saturated. During this first phase, pressure gradients are progressively building up. The hillslope dynamic is dominated by subsurface pressure flows: the old water being pushed out of the riparian zone by the new entering rainfall water (Fig. 4). The flood flow is mainly composed of old water coming from the riparian zone (Fig. 3-b). The second phase corresponding to a rapid extension of the saturated areas only begins after 20 hours.

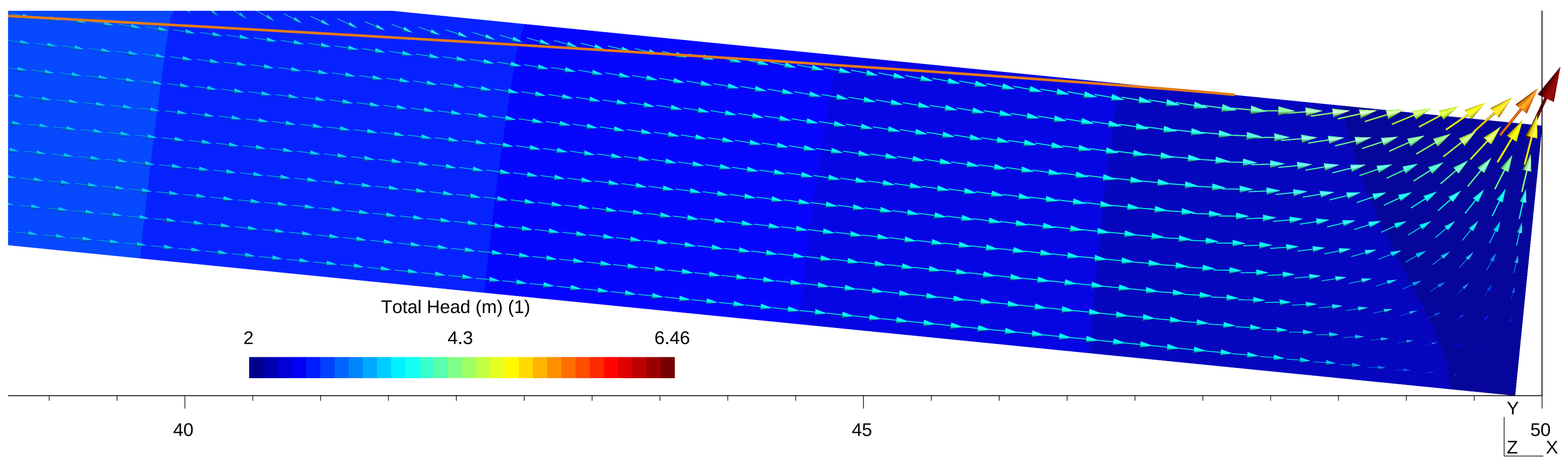

Fig. 4. Total pressure head network and fluxes in the hillslope after 10h, i.e. a 300 mm rainfall amount (with a zero-flow condition applied at the downstream boundary).

Conclusion

The simulations confirm the generally high observed old water contribution to stormflows. It also reveals the major impact of the riparian zones on the rainfall-runoff dynamics, zones which are never really well described in hydrological models.

IV. Comparison with hsB

The hillslope storage Boussinesq [4] is now a popular simplified model used to analyse the rainfall-runoff dynamics.

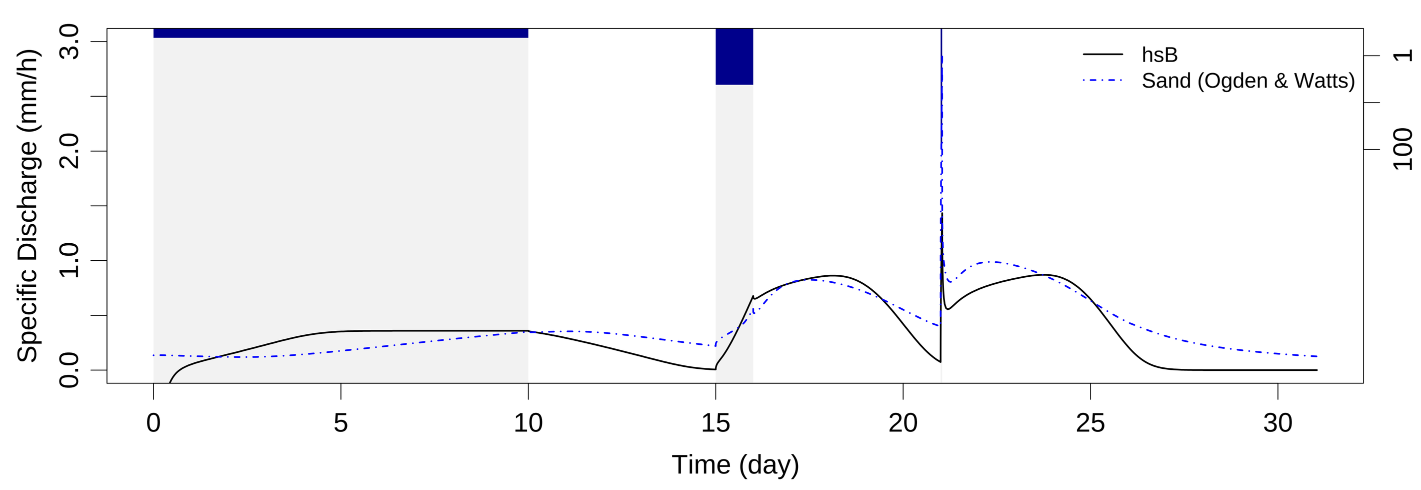

Fig. 5. Comparison of the hsB and 3-D coupled surface-subsurface models on a non-convergent hillslope (with a zero-flow condition applied at the downstream boundary).

The simulated outflow of the simplified hillslope receiving 3 successive 100 mm/h rainfall bursts computed with hsB and the complete 3-D coupled model were compared (Fig. 5).
The simulated hydrographs have very different shapes with longer decays for the 3-D model and unrealistic flood peaks occurring 3 days after rain cease for hsB. This storms succession scenario reveals the importance of the non-saturated parts of the hillslope, not taken into account in hsB.

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